E117 Exam 1 Rubric (F2015)

In order to receive credit, the following items must appear in your solution in some form:

- Question 1 [65 pts]
  1. Solution Basis $u = e^{px}$ [10 pts]
  2. Characteristic equation $p^3 = -1$ [5 pts]
  3. Definition of p: [10 pts]
     - $p = re^{i\phi}$ OR
     - $p = r(\cos\theta + i\sin\theta)$ OR
     - $p = x + iy$
  4. Substitute p into $p^3 = -1$: [5 pts]
     - $p^3 = r^3 e^{3i\phi} = -1$
     - $p^3 = r^3(\cos 3\theta + i\sin 3\theta) = -1$
     - $p^3 = x^3 - 3xy^2 + 3ix^2y - iy^3 = -1$
  5. Equate real and imaginary parts: [10 pts]
     - $\cos 3\phi = -1, \sin 3\phi = 0$
     - $\cos 3\theta = -1, \sin 3\theta = 0$

       Note that for the two items above, r must equal 1. If this statement is absent, -5pts
     - $x^3 - 3xy^2 = -1, 3x^2y - y^3 = 0$
  6. Three values of p: $p = -1, \frac{1}{2} \pm \frac{i\sqrt{3}}{2}$ [10 pts]

     Note that if one value of p is missing, deduct 5pts. If two or three are missing, zero credit for this part.
  7. Explanation about how to arrive at real solution [10 pts]
     - Complex conjugate coefficients of complex conjugate solution pair results in real solution.
     - $\sin\theta$ and $\cos\theta$ are linearly independent solutions.

     If they state an explanation that is almost right, but not quite, deduct 5 pts
  8. Final solution[5pts]

     $u(x) = c_0 e^{-x} + e^{\frac{x}{2}}(c_1 \cos(\frac{x\sqrt{3}}{2}) + c_2 \sin(\frac{x\sqrt{3}}{2}))$

  A few students used Argand diagrams in place of items 3 - 5. If they did so in a way that demonstrates knowledge about how to do other problems, then they receive full credit for these items. If, however, they do so in reference to a previously solved problem, then they have not demonstrated that they could solve an arbitrary problem of the same nature, and 10 points will be deducted from these items.

  If they have $\frac{-1}{2}$ instead of $\frac{1}{2}$, then deduct 5 pts.

- Question 2 [65 pts]

  1. Correct figure for part a [5 pts]
  2. Correct figure for part b [5 pts]
  3. Correct final answer for part a ($f(x) = 1$) [15 pts]
  4. For part b, $a_n = 0$ with explanation [5 pts]
  5. Correct setup of integration: [10 pts]
     - $b_n = \int_{-1}^{1} f_{odd} \sin n\pi x dx = 2\int_0^1 \sin n\pi x dx$ (5 pts each) OR
     - $b_n = 2\int_0^1 \sin n\pi x dx$ OR
     - $b_n = \int_{-1}^{0} \sin n\pi x dx + \int_0^1 - \sin n\pi x dx$
  6. Correct integration [10 pts]
  7. Evaluating the expression resulting in different values for even and odd values of n [10 pts]
  8. Final solution [5 pts]

     Note for the final solution, if k is initialized incorrectly, then no credit is given for final solution. However, also note that if $n = 2k+1$, then k should be initialized at 1.

     If part b is completely wrong, but part a follows the same pattern, then swap how the parts are graded.

     If they do not evaluate the expression to find different values for even and odd n, then they can still receive credit for inserting their expression into a final solution.

- Question 3 [70 pts]

    1. Assumed form of solution: $T = A(t)B(x)$ [5 pts]
    2. Separate variables: $\frac{\dot{A}}{A} = \frac{B''}{B}$ [10 pts]
    3. Identify separation constant: $\frac{\dot{A}}{A} = \frac{B''}{B} = -\lambda$ [5 pts]
    4. General solution for B: $B = c_0 \cos \sqrt{\lambda} x + c_1 \sin \sqrt{\lambda} x$ [10 pts]
    5. Apply BC at $x = 0$ to get $c_0 = 0$ [5 pts]
    6. Apply BC at $x = 1$ to get $\sin \sqrt{\lambda} = 0$ [5 pts]
    7. Find $\lambda = n^2 \pi^2$ [5 pts]
    8. General solution for A: $A = c_2 e^{-n^2 \pi^2 t}$ [5 pts]
    9. General solution or basis solution for T: [5 pts]
       $T = \sum_{n=1}^{\infty} b_n e^{-n^2 \pi^2 t} \sin n\pi x$ or $T_n = b_n e^{-n^2 \pi^2 t} \sin n\pi x$

       If a student states a basis solution without an arbitrary constant, full credit is given.
    10. Equate general solution evaluated at $t = 0$ to initial condition [5 pts]
    11. Final solution [5 pts]